Highly computable graphs and their domatic numbers

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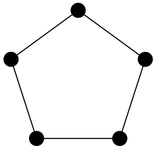
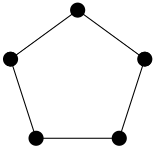
Domatic Partitions

Domatic 2-partition



No domatic 3-partition



Domatic number of a graph G:
$d(G) =$ the max n s.t. G has a domatic n-partition

Computable domatic number of a graph G:
$d^c(G) =$ the max n s.t. G has a computable domatic n-partition

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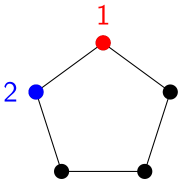

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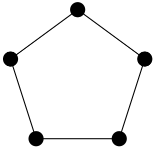

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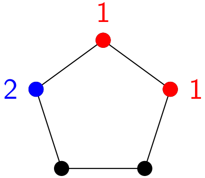

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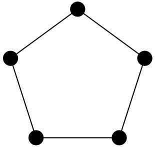

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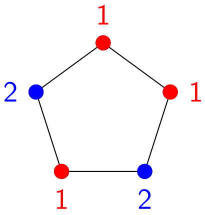

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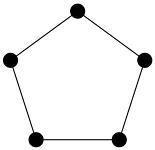

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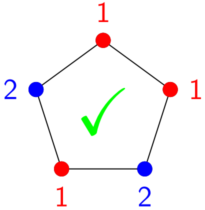

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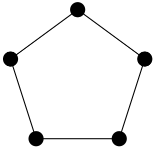

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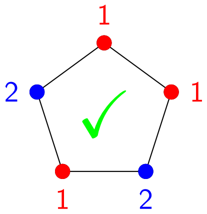

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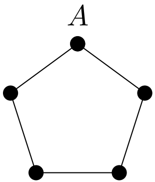

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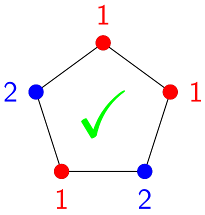

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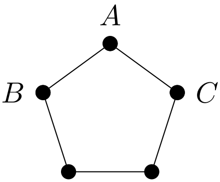

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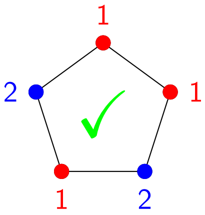

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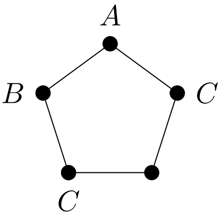

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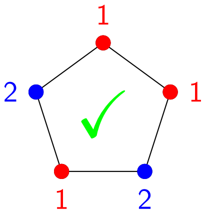

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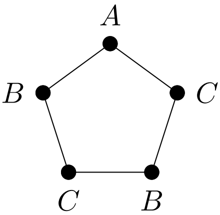

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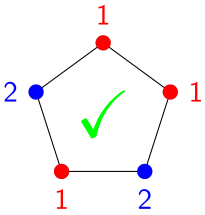

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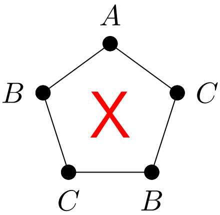

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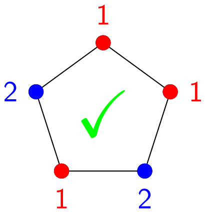

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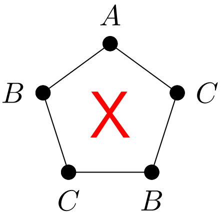

$d(G) = 2$

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Highly Computable Graphs

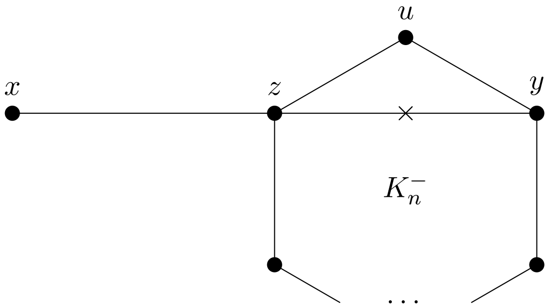
Definition

A graph $G = (V, E)$ is **highly computable** if V and E are computable sets (i.e., sets whose membership functions are computable) and there is a computable function that, when given $v \in V$, outputs the degree of v.

Theorem (Jura, Levin, M.)

For every $n \geq 3$, there is a highly computable graph G such that $d(G) = n$ and $d^c(G) = n - 1$.

The K_n^--Gadget



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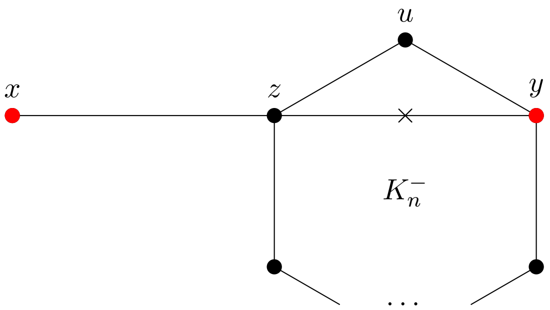
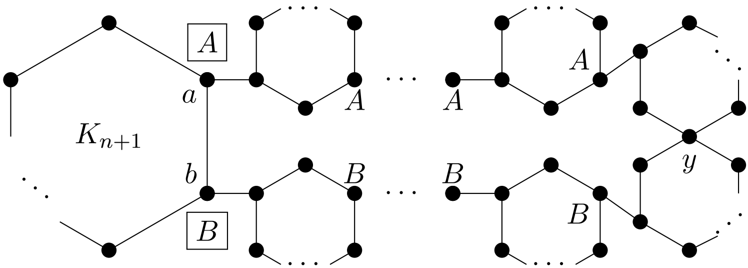


$d^c(G) < n$

Figure : Trapping a purported computable domatic n-partition.



$$d(G) = n$$

Figure : A domatic n-partition of the trap.