There Are Two Errors in the the Title of This Talk

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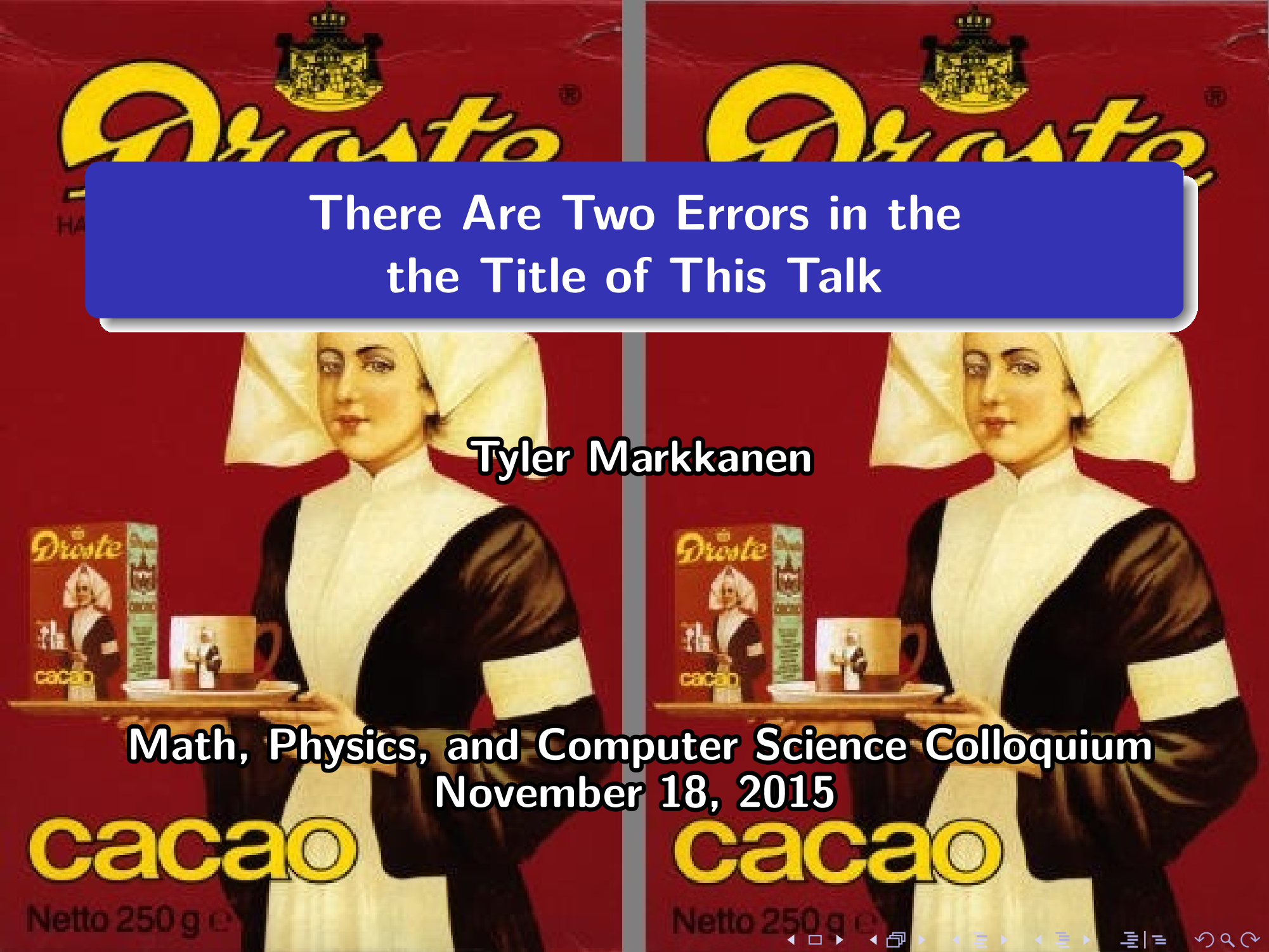

1. The tooth fairy is real.
2. Both of these sentences are false.

This sentence is false.

The set of all sets that are not elements of themselves

The smallest natural number not
definable in less than twelve words

John is reading this sentence.

This sentence is false.

The set of all sets that are not elements of themselves

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The set of all sets that are not elements of themselves

The smallest natural number not
definable in less than twelve words

John is reading this sentence.

Definition

The **quotation** of P is

$$\text{“}P\text{”}$$

Example

Let P be:

Hello World!

The quotation of P is:

“Hello World!”

1. John is reading “John is reading”
2. John is reading “John is reading “John is reading””
   (This only says John is reading sent. 1, not sent. 2.)
3. John is reading “John is reading “John is reading . . .

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Definition

The **norm** of P is

$$P\text{“}P\text{”}$$

Example

Let P be:

Math is fun

Then the norm of P is:

Math is fun “Math is fun”

Consider the following sentence:
John is reading the norm of "John is reading the norm of"

Why it says that John is reading that very sentence:

The norm of

John is reading the norm of

is

John is reading the norm of "John is reading the norm of"

Exercise

Write a sentence asserting that John is reading the norm of that very sentence.

Notation Help

J	John is reading
J“J”	John is reading “John is reading”
N“J”	the norm of “John is reading”
JN“JN”	John is reading the norm of “John is reading the norm of”

Machines & Self-Reference

- Consider a machine M that prints out expressions made with five symbols:
$$\sim, \quad P, \quad N, \quad (, \quad \text{and} \quad).$$
- An **expression** is any non-empty finite string of symbols, e.g.,
$$N\sim(P, \qquad)P((((((((, \qquad \text{and} \qquad P(N(\sim)).$$
- For an expression X, a **sentence** is any expression of the form:
$$P(X), \quad PN(X), \quad \sim P(X), \quad \text{or} \quad \sim PN(X).$$
- We interpret the meaning of the symbols as follows.
P: "is printable"
$\sim$: "not"
N: "the **norm** of," e.g., the norm of $P\sim$ is $P\sim(P\sim)$

Telling the Truth

Rule:
The machine M can only print *TRUE* sentences.

Example

1. If M prints $P(X)$, then X is printable. So M will print X.
2. If M prints $\sim PN(X)$, then the norm of X, i.e., $X(X)$, is not printable. So M never prints $X(X)$.
3. If M prints X, then M does not necessarily print $P(X)$.

Question:
Can M print *EVERY* true sentence?

You Can't Handle the Truth!

Can such an M print *ALL* true sentences?

No. The following sentence is true but M will not print it:

$$\sim PN(\sim PN)$$

This is because of the following fact:

$$\sim PN(\sim PN) \text{ is true } \iff \sim PN(\sim PN) \text{ is not printable}$$

The 1# Programming Language on Register Machines

Designed by Larry Moss, Indiana University Bloomington

Exercise

Write a sentence asserting that John is reading the norm of that very sentence.

Exercise

Write a sentence asserting that John is reading the norm of that very sentence.

Answer

$JNN\text{“}JNN\text{”}$

The norm of

$$JNN$$

is

$$JNN\text{“}JNN\text{”} \tag{1}$$

So our original sentence asserts that John is reading the norm of (1), i.e., the norm of our original sentence.

Exercise

Write an expression that names its own norm.

Exercise

If Q means *the quotation of*, write two different sentences X and Y so that each names the other.

Thank you.


Robert M. Martin.
There Are Two Errors in the the Title of This Book, Revised and Expanded (Again): A Sourcebook of Philosophical Puzzles, Problems, and Paradoxes, 3/e.
Broadview Press, 2011.


Lawrence S. Moss.
1#: a text register machine introduction to computability theory, 2015.


Robert I. Soare.
Turing Computability: Theory and Applications.
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